31/05/19

**THM 121 Business Mathematics**

**Final Exam**

**Note to the students**:

* Calculations to reach your answers shall be thoroughly shown. Otherwise, questions will NOT be graded.
* You can use a calculator throughout the exam.
1. Write the equation of the line with the given properties: (**3** Points)

Passes through the point (4, 1) and is perpendicular to the line 2χ + 3y = 6.

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1. A credit card company estimates that the average cardholder owed $ 8,053 in the year 2014 and $ 9,333 in 2018. Suppose average cardholder debt *D* grows at a constant rate.
2. Express *D* as a linear function of time *t*, where *t* is the number of years after 2014. Draw the graph of this function. (**4** Points)
3. Use the function in part a) to predict the average cardholder debt in the year 2027. (**1** Point)

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1. Approximately when will the average cardholder debt be double the amount of that in year 2014? (**2** Points)

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3) A manufacturer estimates that each unit of a particular commodity can be sold for $ 4 more than it costs to produce. There is also a fixed cost of $ 20,000 associated with the production of the commodity.

a) Express total profit *P*(χ) as a function of the level of production χ. (**2** Points)

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b) How much profit (or loss) is generated when χ = 6,000 units are produced? When χ = 24,000? What is the smallest number of units that must be sold for production to be profitable? (**2** points)

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1. Find the average profit function *AP*(χ). What is the average profit when 12,500 units are produced? (**2** Points)

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1. Producers will supply χ units of a certain commodity to the market when the price is *P* = *S*(χ) dollars per unit, and consumers will demand (i.e. buy) χ units when the price is *P* = *D*(χ) dollars per unit, where:

***S*(χ) = 3 χ + 25 and *D*(χ) = 400 / (2χ + 1)**

1. Find the equilibrium production level **χe** and the equilibrium price **Pe**. (**2** Points)

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1. Draw the supply and demand curves on the same graph. (**3** Points)
2. Where does the supply curve cross the y axis? Describe the economic significance of this point? (**2** Points)

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1. Find the indicated limit of the following function (if it exists): (**3** Points)

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1. The concentration of a drug in a patient’s bloodstream *t* hours after an injection is C(*t*) milligrams per milliliter where:

**C(*t*) = (0.50 / (2t2 + 1)) + 0.011**

1. What is the concentration of drug immediately after the injection (i.e. when t = 0)? (**1** Point)

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1. By how much does the concentration change during the 5th hour? Does it increase or decrease over this time period? (**2** Points)

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1. What is the residual concentration of drug, that is, the concentration that remains in the long run (as *t* → ∞)? (**2** Points)

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1. List all the values of χ for which the given function is not continuous. (**2** Points)

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1. In certain situations, it is necessary to weigh the benefit of pursuing a certain goal against the cost of achieving that goal. For instance, suppose that to remove χ% of the pollution from an oil spill, it costs *C* thousands of dollars, where:

***C*(χ) = 14χ / (100 – χ)**

1. How much does it cost to remove **25 %** of the pollution? **50%**? (**2** Points)

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1. Sketch the graph of the cost function C. (**3** Points)
2. What happens as **χ → 100-**? Is it possible to remove all the pollution? Why? Why not? (**2** Points)

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**N.B**. Round your answers to the **nearest cent** for questions 2, 3, 4, 6 & 8.

**GOOD LUCK!**